Advancing a More Accurate Third Derivative Numerical Method to Enhance the Solution of Food Security Problems in Developing Countries

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Abstract
Food security has to do with food availability and quality control which are innovations in food measurement and monitoring. Numerical method is a very important tool for solving problems of food security since such problems have to do with rate of change which resolves to differential equations. This paper describes the determination of a more accurate numerical method to enhance the solution of food security problems in developing countries. Euler's method, Trapezoidal rule, Simpon's rule and one - step third derivative method were used to solve two initial value problems of first order ordinary differential equations to determine a method with better accuracy that will be recommended for solution of practical problems like food security. Comparison of the numerical results with the exact solution showed that the one - step third derivative method has better accuracy than the other methods examined and thus recommended for solving food security problems having to do with rate of change.

Keywords: *Food security, Numerical method, Differential equation, Developing Countries.*

Introduction

In the words of Jenkins and Scanlan (2001), food is the most basic of human needs and is central to the discussion of human rights and social development. For this reason, developed and developing countries make considerable efforts to increase their food production capacity to ensure there is an adequate quantity of available food and that the citizens take balanced diets so as to positively affect the overall economic development of the countries. (Macnamara,1973).

Food security in its most basic form is defined as the access to all people to the food needed for a healthy life at all times (Eide, 1999). In a simple language, a country is food-secure when majority of its population have access to food of adequate quantity and quality consistent with decent existence at all times (Rentlinger, 1983 and Idachaba, 2004).

Food security exists when all people, at all times, have physical and economic access to sufficient, safe and nutritious food that meets their dietary needs and food preferences for an active and healthy life (World Food Summit, 1996).

In addition to food availability, food hygiene and safety are essential in order to protect the health of the people. Therefore health and safety consideration becomes important in food production.

Factors that are responsible for the precarious food insecurity in Africa are: low agricultural productivity, lack of agricultural policies, poor infrastructure and high – transport costs, lack of appropriate marketing strategies, frequent extreme weather events, high – disease burden including HIV/AIDS, weak financial support systems, lack of safety net systems and political conflicts, (Haile, 2005). According to Smith, (1998), the greatest challenge facing food security in Africa is poverty.

According to Ojo and Adebayo (2012), Nigeria is one of the food-deficit countries in sub-saharan Africa although it is arguably better in terms of production than the others.

According to Akinyosoye, (2007), Nigerian government has embraced the idea of using biotechnology to boost food production as a pre-condition for food security. Furthermore, Nigerian agricultural scientists have been very enthusiastic in advancing the frontier of knowledge in biotechnology.

Many problems of food security have to do with rate of change, for example: determining the ratio of the quantity of food available to the number of consumers, size reduction in milling machines, heat transfer in drying process and so on.

These problems will resolve into differential equations and numerical methods have been found as effective tools for solving such problems, Famurewa and Olorunsola (2014). Examples of such numerical methods are: Euler's method, Trapezoidal rule, Simpon's rule, Runge - Kutta method, Adams' explicit and implicit methods, Backward Differentiation Formula, implicit multiderivative methods, to mention but a few. This research therefore intends to advance a more accurate numerical method that would enhance the solution of food security problem

Methodology

In this work, Euler's method of the form:

$$y_{n+1} = y_n + hf_n \qquad (1)$$

Trapezoidal rule of the form:

$$y_{n+1} = y_n + \frac{h}{2}(f_{n+1} + f_n) \qquad (2)$$

Simpon's rule of the form:

$$y_{n+1} = y_n + \frac{h}{3}(f_{n+2} + 4f_{n+1} + f_n) \qquad (3)$$

and One - step third derivative method of the form:

$$y_{n+1} = y_n + \frac{h}{2}(y_{n+1}^{1} + y_n^{1}) - \frac{h^2}{10}(y_{n+1}^{11} - y_n^{11}) + \frac{h^3}{120}(y_{n+1}^{111} + y_n^{111}) \qquad (4)$$

will be used to solve the two sampled initial value problems of first order ordinary differential equations.

The test problems are:

(1) $y^1 = x + y,\ \ y(0) = 1\ with\ h = 0.1$

Exact solution is $y(x) = 2e^x - x - 1$

(2) $y^1 = -y,\ \ y(0) = 1\ with\ h = 0.1$

Exact solution is $y(x) = e^{-x}$

Results and Discussion

The results and errors obtained from the exact solution and the numerical methods are presented in Tables 1 - 4. It was observed that all the methods are good but the most accurate is the one - step third derivative method because it is the closest to the exact solution, this is a proof of the accuracy of the method to solve food security problems that resolve to ordinary differential equation.

Table1: The result obtained for problem 1

EXACT-SOLUTION	EULER'S METHOD	TRAPEZOIDAL RULE	SIMPSON'S RULE	THIRD DERIVATIVE METHOD
1.1103418361512953	1.1000000000000001	1.1100000000000000	1.1100000000000001	1.1103418361514121
1.2428055163203395	1.2200000000000002	1.2420500000000001	1.2426666666666666	1.2428055163205978
1.3997176151520065	1.3620000000000001	1.3984652500000001	1.3991466666666668	1.3997176151524342
1.5836493952825408	1.5282000000000000	1.5818041012500001	1.5825570666666668	1.5836493952831714
1.7974425414002564	1.7210200000000001	1.7948935318812502	1.7957255586666667	1.7974425414011277
2.0442376007810177	1.9431220000000000	2.0408573527287817	2.0417767423266668	2.0442376007821732
2.3275054149409531	2.1974342000000000	2.3231473747653038	2.3241633002709667	2.3275054149424426
2.6510818569849350	2.4871776200000000	2.6455778491156607	2.6467004467994188	2.6510818569868166
3.0192062223138993	2.8158953820000000	3.0123635232728052	3.0136039937133576	3.0192062223162388
3.4365636569180902	3.1874849202000000	3.4281616932164498	3.4295324130532605	3.4365636569209630

Table 2: The result obtained for problem 2

EXACT-SOLUTION	EULER'S METHOD	TRAPEZOIDAL RULE	SIMPSON'S RULE	THIRD DERIVATIVE METHOD

0.9048374180359595	0.9000000000000000	0,9050000000000000	0.9050000000000000	0.9050841666666667
0.8187307530779818	0.8100000000000001	0.819025000000000	0.8186666666666667	0.8191523137588643
0.7408182206817178	0,7290000000000001	0.741217625000000	0.7408933333333334	0.7413540889017383
0.6703200460356393	0.6561000000000001	0.670801950625000	0.6705084666666666	0.6709196273358526
0.6065306597126334	0.5904900000000001	0.6070757653156250	0.6068101623333333	0.6071519335975062
0.5488116360940265	0.5314410000000001	0.5494035676106407	0.5491631969116666	0.5494199842888023
0.4965853037914095	0.4782969000000000	0.4972102286876298	0.4969926932050583	0.4971524836848332
0.4493289641172216	0.4304672100000000	0.4499752569623050	0.4493289641172215	0.4498322103855900
0.4065696597405992	0.3874204890000000	0.4072276075508861	0.4070494405522729	0.4069908990689931
0.3678794411714423	0.3486784401000000	0.3685409848335519	0.3683797436998070	0.3682046066966087

Table 3: The error obtained for problem 1

EULER'S METHOD	TRAPEZOIDAL RULE	SIMPSON'S RULE	THIRD DERIVATIVE METHOD
1.034184e-002	3.418362 e-004	3.418362 e-004	1.167955e-013
2.280552 e-002	7.555163 e-004	1.388497 e-004	2.582379e-013
3.771762 e-002	1.252365 e-003	5.709485 e-004	4.276579e-013
5.544940 e-002	1.845294 e-003	1.092329 e-003	6.306067e-013

7.642254 e-002	2.549010 e-003	1.716983 e-003	8.713030e-013
1.011156 e-001	3.380248 e-003	2.460858 e-003	1.155520e-012
1.300712 e-001	4.358040 e-003	3.342115 e-003	1.489475e-012
1.639042 e-001	5.504008 e-003	4.381410 e-003	1.881606e-012
2.033108 e-001	6.842699 e-003	5.602229 e-003	2.339462e-012
2.490787 e-001	8.401964 e-003	7.031244 e-003	2.872813e-012

Table 4: The error obtained for problem 2

EULER'S METHOD	TRAPEZOIDAL RULE	SIMPSON'S RULE	THIRD DERIVATIVE METHOD
4.837418 e-003	1.625820 e-004	1.625820 e-004	2.467486e-004
8.730753 e-003	2.942469 e-004	6.408641 e-005	4.215607e-004
1.181822 e-002	3.994043 e-004	7.511265 e-005	5.358682e-004
1.422005 e-002	4.819046 e-004	1.884206 e-004	5.995813e-004
1.604066 e-002	5.451056 e-004	2.795026 e-004	6.212739e-004
1.737064 e-002	5.919315 e-004	3.515608 e-004	6.083482e-004
1.828840 e-002	6.249249 e-004	4.073894 e-004	5.671799e-004
1.886175 e-002	6.462928 e-004	4.494232 e-004	5.032463e-004
1.914917 e-002	6.579478 e-004	4.797808 e-004	4.212393e-004
1.920100 e-002	6.615437 e-004	5.003025 e-004	3.251655e-004

Conclusion

In this study, Euler's method, Trapezoidal rule, Simpon's rule and One - step third derivative methods were used to solve two sampled problems of first order ordinary differential equations in order to determine the method with better accuracy. The results and errors obtained showed that the one - step third derivative method is more accurate, effective and efficient.

The one - step third derivative method of the form:

$$y_{n+1} = y_n + \frac{h}{2}(y^1_{n+1} + y^1_n) - \frac{h^2}{10}(y^{11}_{n+1} - y_n^{11}) + \frac{h^3}{120}(y^{111}_{n+1} + y_n^{111})$$

can therefore be adopted for solving problems of food security and other security problems that have to do with rate of change.

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